# **Decoding Algorithms**

Deciding under Uncertainty in Machine Translation



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https://probabll.github.io

This class is about the (decoding) algorithms that turn input text in one language into output text in another, with the help of a (language) model to handle the many choices along the way.

#### This class is for those

- developing new algorithms
- choosing amongst existing algorithms
- using decoding algorithms

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1. NMT

- 2. Translating
  - Samplers
  - **Decision Rules**

3. Modern Decoding, as I see it

# **NMT**

# The autoregressive language model API

Throughout the talk, I assume that one's preferred MT engine is powered by an *autoregressive language model*.

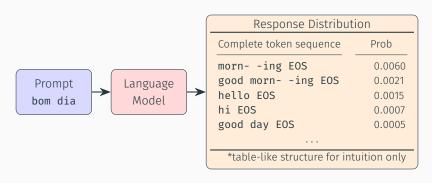
This choice implies access to a specific API that makes various crucial operations (incl. those needed for training and decoding) feasible to varying degrees of approximation.

This API allows us to regard an LM as a means to predict *conditional* (that is, input-specific) *probability distributions* (cpds).<sup>1</sup>

<sup>&</sup>lt;sup>1</sup>You may also reason the other way around: LMs are designed to predict input-specific probability distributions, when they are designed to comply with a certain API, they are regarded as *autoregressive*.

### $\mathsf{Prompt} o \mathsf{Language} \ \mathsf{Model} o \mathsf{Distribution} \ \mathsf{over} \ \mathsf{Responses}$

From sufficiently far away, we can regard an LM as machine that maps any one prompt to a prompt-specific *probability distribution* whose outcome space is the set of all complete token sequences.



# Short Digression: Statistical Learning

Training algorithms that approximate maximum likelihood estimation (e.g., supervised tuning or fine tuning using translation data) will make these cpds 'more coherent' with statistics of observed *translation data*.

That's because LMs trained like that learn to predict distributions from *data samples* (not from those samples' 'probabilities').

Roughly, the more training data you observe, the less you can tell data samples from model samples apart.<sup>2</sup>

Prompt bom dia

LM-Sampled Responses

good morning

hello

morning!

hi there!

morning

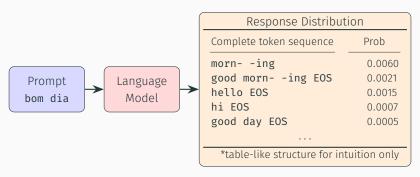
Responses by Human Translators

morning!
good morning
hello
good morning
hey there!

<sup>&</sup>lt;sup>2</sup>The notion of 'sample' here is a rather specific one, we will talk about it later.

# Not quite the whole story...

As we zoom in, we realise that an LM does not really build anything like this 'tabular' representation of the cpd:

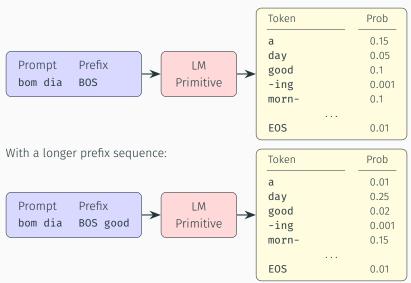


rather, it parameterises a special kind of iterative process, which *implicitly* identifies one such object.<sup>3</sup>

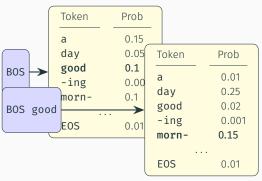
<sup>&</sup>lt;sup>3</sup>Then, certain ways of interacting with that iterative process is statistically equivalent to interacting with the table-like thing.

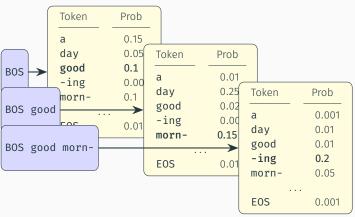
### Prompt and Prefix ightarrow LM Primitive ightarrow Next-Token Distribution

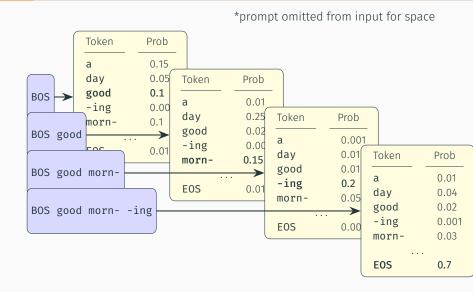
With an empty prefix (represented by a sequence containing BOS only)

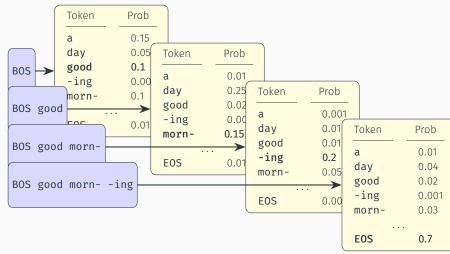












#### **Factorised Probabilities**

Given a prompt x, an autoregressive LM factorises the probability it assigns to any one outcome sequence  $y = \langle y_1, \dots, y_\ell \rangle$  along the  $\ell$  tokens that make up the outcome, as follows:

$$p_{\theta}(y|x) = \prod_{i=1}^{\ell} p_{\theta}(y_i|x, y_{< i}).$$
 (1)

<sup>&</sup>lt;sup>5</sup>Mapping from  $(x, y_{< i})$  to one such vector is a task easily accomplished by architectures like RNNs and Transformers.

#### **Factorised Probabilities**

Given a prompt x, an autoregressive LM factorises the probability it assigns to any one outcome sequence  $y = \langle y_1, \dots, y_\ell \rangle$  along the  $\ell$  tokens that make up the outcome, as follows:

$$p_{\theta}(y|X) = \prod_{i=1}^{\ell} p_{\theta}(y_i|X, y_{< i}).$$
 (1)

Under the assumption that the vocabulary is finite, with *V* symbols, and independent of the position *i*, any one of the next-token distributions is specifiable by a *V*-dimensional probability vector.<sup>5</sup>

<sup>&</sup>lt;sup>5</sup>Mapping from  $(x, y_{< i})$  to one such vector is a task easily accomplished by architectures like RNNs and Transformers.

# Why are LMs so often Designed this Way?

#### There are various answers, here are some

- 1. there are infinitely many responses, but only finitely many tokens at each step;
- 2. this allows us to assess the probability mass of a response efficiently;
- 3. this allows us to 'draw' outcomes from the model, often with useful statistical guarantees.
- (1) is about feasibility, (2) is useful for supervised training (but also some forms of decoding), (3) is particularly useful for decoding (but also some forms of training).

### Summary

We can regard an LM as a mechanism trained to predict entire input-specific probability distributions over the space of responses.

The most common such mechanisms (incl. encoder-decoder and decoder-only Transformer models) are built upon a chain-rule factorisation of the probability of sequences. This allows us to regard LMs as offering 4 features (the first 2 being the primitives):

- 1. given prompt x and prefix r, assign probability p(t|x,r) to token t
- 2. given x and r, draw a token t with probability p(t|x,r)
- 3. assign probability p(y|x) to a response y given x
- 4. with probability p(y|x), draw a response y given x

There are interesting designs that violate this API (e.g., EBMs), but I am not covering those today.

**Translating** 

#### Do Translation Models Translate?

"By what built-in mechanism may the model autonomously decide that a response *y* is to be regarded as the translation of *x*?"

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Our models do not have the agency do decide. But we can design recipes—which our models do parameterise—to automate decision making. Those recipes are called *decoding algorithms*.

# **Principles**

Let's outline basic principles we would like a decoding algorithm to observe

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- translations should be in some sense preferred by the model (else, what is the difference between using one model or another?);
- 2. translations are ideally good for their prompts, let's say they ought to be *adequate*;

#### Do Translation Models Prefer Some Translations to Others?

In one sense, our models are not very picky. So long as the individual tokens are known 'a what the cat xxx? EOS' is in the outcome space of any model no matter the prompt.

That said, given the prompt `olha, um gato!`, two models that share the same vocabulary may easily differ in the probabilities they assign to 'look, a cat! EOS' and 'a what the cat xxx? EOS'.

We can regard probabilities as expressing a notion of preference that's 'native' to the model.<sup>6</sup>

<sup>&</sup>lt;sup>6</sup>This notion is in fact coherent with most forms of training, where model parameters are chosen to assign high probability to observed data.

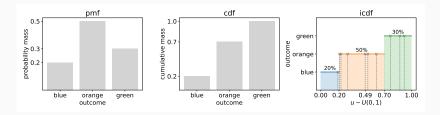
# Translating Samplers

# **Unbiased Sampling**

A sampler draws realisations of a random variable. For us, this means drawing responses (i.e., token sequences that end in EOS) from the distribution that an LM (implicitly) predicts when given a prompt x.

An unbiased sampler is one where, if we draw N samples independently of one another, the relative frequency of any of the sampled responses is an unbiased estimator of that response's probability under the model, and the estimation variance decays as N increases.

# Example: sampling from a distribution over 3 categories



From the probability mass function (pmf) we obtain the cumulative distribution function (cdf), we then characterise the cdf's inverse (icdf). The icdf associates each outcome with a line segment whose length equals the outcome's probability mass.

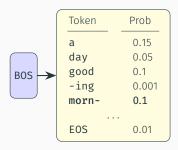
The icdf transforms a uniform random generator into an unbiased sampler for this distribution. The example shows 10 samples (e.g., 0.48 maps to **orange**, as do all numbers between 0.2 and 0.7).

# Ancestral (or Forward) Sampling

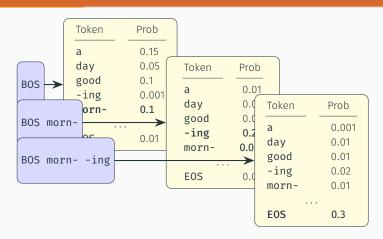
As a consequence of the API we agreed upon, a simple iterative algorithm can be shown to result in unbiased samples from the distribution over *responses*:<sup>7</sup>

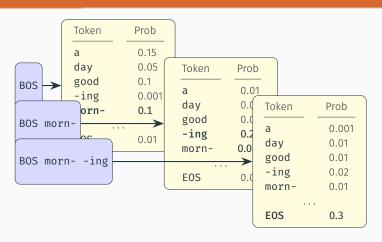
- 1. Reset the sampler state (i.e., condition on prompt and start an empty generation prefix).
- Use the LM to obtain the next-token distribution, draw the next token from it (via the icdf method) and extend the generation prefix with it.
- 3. If the token was EOS, terminate the algorithm returning the sampled sequence, else repeat from (2).

<sup>&</sup>lt;sup>7</sup>[1, 19]







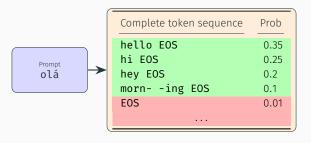


Return morn- -ing EOS with  $p_{\theta}(\text{morn--ing EOS}|\text{bom dia}) = 0.1 \times 0.2 \times 0.3 = 0.006$ 

# A Critical Eye

Unbiased sampling operationalises a notion of 'preferred under the model', but this notion is a 'statistical' one: the decisions it can support get increasingly risky the less samples we draw.

If we collect many samples, we expect a fraction to come from the red group (1 in 10, on average).



But, if we draw one sample, it might well be **EOS** or one of the outcomes in that group.

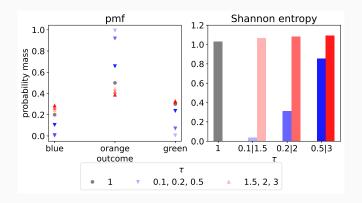
# **Biased Samplers**

A *biased* sampler deviates from the model's native probabilistic interpretation.

But, if we invest so much in model training, are there good reasons for deviating from the model?

# Biasing a sampler with a 'temperature'

Let's use the 3 categories example. Say the probabilities are  $p_1, p_2, p_3$ . We can define alternative distributions by introducing a 'temperature parameter'  $\tau > 0$ : then new pmfs can be obtained via  $\frac{p_k^{1/\tau}}{p_1^{1/\tau} + p_2^{1/\tau} + p_3^{1/\tau}}$ 



# **Temperature Sampling**

A modification of ancestral sampling, where we transform next-token distributions by exponentiation and renormalisation *before* sampling.

## Critical Eye - Choosing the Temperature

Should we aim for more or less entropy?

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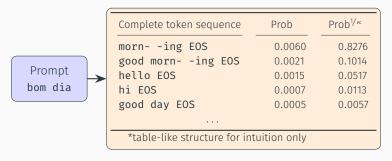
In the literature, you will find advices such as

- · more entropy, as to promote diversity amongst samples;
- · less entropy, as to discourage/prune low-probability outcomes.

There is no reason to believe that we can motivate a choice of  $\tau$  from introspection alone. The best we can do is to treat  $\tau$  as a hyperparameter and pick it experimentally (under the assumption that we can simulate the test conditions reasonably well in the lab).

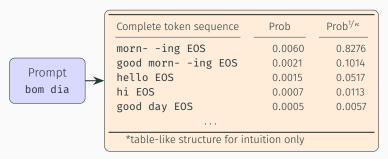
### Critical Eye - Understanding the Effect

By applying a temperature (say  $\tau=0.5$ ) to each next-token distribution from left-to-right, are we essentially applying a temperature  $\kappa$  to the distribution over responses?



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By chain rule of probabilities, we know that there exists an autoregressive decomposition of  $\operatorname{Prob}^{1/\kappa}$  along the token sequence, but that factorisation **is not** of the form  $\propto \prod_{i=1}^\ell p_\theta^\tau(y_i|x,y_{< i})$ , where we simply exponentiate and normalise the original next-token cpds.

### Critical Eye - Summary

We cannot expect a temperature to serve all prompts alike.

We need to treat temperature as a hyperparameter.

The intuition we developed in the simple case of distributions over categories does not transfer to distributions over sequences.

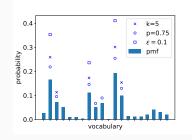
## Probability- and Mode-Seeking Samplers

When we sample with low temperature, we not only discourage the outcomes with less mass, we exaggerate differences throughout the whole probability spectrum, distorting the shape of the distribution.

Other ideas are formulated more directly as a form of pruning and tend to better preserve the relative merits of the outcomes that are not pruned.

## Truncation Sampling: top-k, top-p, and $\epsilon$ -sampling.

Choose a criterion, prune outcomes that do not meet it, renormalise the next-token cpd, sample.



The top-k sampler [7] prunes all but the k most probable tokens, the next-token cpd is then renormalised over this reduced outcome space.

The top-p sampler [aka nucleus sampler; 14] also prunes all but the most probable tokens, but it keeps as many tokens as needed to cover a pre-specified amount of probability mass.

The  $\epsilon$ -sampler [13] prunes any outcome whose mass is less than some  $\epsilon > 0$ .

### Critical Eye

These biased samplers operationalise a clearer bet: we are betting that good sequences will have few, if any, low-probability tokens.

The question is, why should that be the case?

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The question is, why should that be the case?

- there is no obvious reason why we should expect a good sequence to have no (or even very few) low-probability tokens;
- we may like models that exhibit such a property, but ours were not designed and trained to meet it.

## **Locally Typical Sampling**

Meister et al. [26] motivate a different criterion to sort the tokens for a nucleus sampler (think of it as defining the nucleus differently).

Keep enough tokens to cover at least a probability mass p, but sort tokens on the absolute difference between their individual 'surprisal'

 $-\log p(t|x,r)$  for a token t, prompt x and generated prefix r

and the expected surprisal (aka Shannon entropy):

$$-\frac{1}{V}\sum_{w\in\mathcal{W}}\log p(w|x,r).$$

## Critical Eye

The original paper motivated locally typical sampling from i) findings in psycholinguistics, and ii) a remarkable property of certain Markov processes (MPs) concerning how surprisal values distribute.

To my understanding, there are at least two points of contention:

- The psycholinguistic finding need not transfer to any one model (we may wish that to be true, but it need not be)
- Autoregressive LMs are not guaranteed to meet the necessary formal properties of MPs that exhibit strong regularities in how surprisals distribute.

Nonetheless, typical sampling offers an interesting, 'non-mode-seeking' way to truncate next-token distributions.

#### Stochastic Processes

When we pair a model and a choice of sampler, we *induce* a distribution over responses [4].

If this sampler is unbiased, the distribution is precisely the one the model predicts.

When the sampler is biased, we cannot say much.

But we can say one thing: relative to that distribution (unless it happens to have a remarkably low entropy), a single sample conveys very little information.

### Summary

Unbiased samplers operationalise a notion of 'preferred by the model': they allow us to interact with the prompt-specific probability distribution that is coherent with our model.

Biased samplers capture preferences that we motivate ourselves (such as more or less entropy, avoiding low-probability transitions, avoiding too-low or too-high token surprisal relative to the entropy of the next-token cpd, etc.).

Samplers induce stochastic processes and it's hard to imagine a property that a single sample is guaranteed to satisfy.

# Translating

Decision Rules: Searching for a Specific Translation

Suppose we could assign a notion of of quality  $\mu(c; x)$  to any candidate translation c of a prompt x.

Example: ask a person to give it a mark, from 0 to 100.

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Wouldn't a good translation be one that maximises that score?

$$y^{\text{decision}} = \operatorname{argmax}_{c \in \mathcal{Y}} \ \mu(c; x)$$
 (2)

This is what we call a *decision rule*, where we search for a specific response, using an explicitly stated criterion.

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How can our translation model be of any use for this? One or both of the following:

- it can contribute to the definition of  $\mu$ ;
- it can prioritises subsets of the search space;

### Most Probable Response

Here's a line of argumentation: "if there is one outcome that my model prefers, that outcome ought to be the mode of the conditional distribution over responses."

$$y^{\text{mode}} = \operatorname{argmax}_{c \in \mathcal{Y}} \ p_{\theta}(c|x) \tag{3}$$

Unlike a single sample from any sampler, this outcome satisfies a clear criterion: its probability is larger than that of any other outcome.

Do you see any problems?

This has come to be known as maximum-a-posteriori (MAP) decoding.

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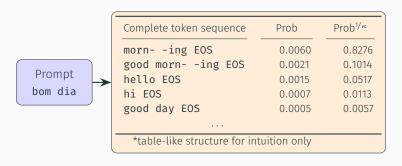
Unlike a single sample from any sampler, this outcome satisfies a clear criterion: its probability is larger than that of any other outcome.

Do you see any problems? I see two: i) go about finding it, and ii) what if the mode is of no special significance?

This has come to be known as maximum-a-posteriori (MAP) decoding.

#### Intractable Search

The search space is unbounded and due to the chain-rule factorisation (no Markov assumptions) dynamic programming isn't possible.



## The Greedy Approximation

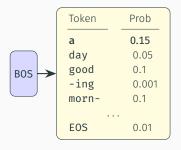
At each step i, we find the token that is assigned maximum probability given the prompt and the generated prefix  $y_{< i}$ :

$$y_i \leftarrow \operatorname{argmax}_{w \in \mathcal{W}} p_{\theta}(w|x, y_{< i})$$
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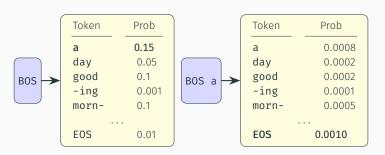
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 (4)



Return a EOS with  $p_{\theta}$  (a EOS|bom dia) = 0.15 × 0.01 = 0.00015

This strategy is simple but makes a lot of search errors (i.e., fails to find the mode).

### Better Approximate Search: Beam Search

At each step, we keep refining a small set of candidates (for example, k = 5 candidates). We could then,

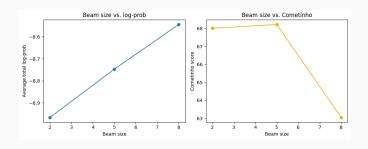
- consider all k × V ways in which these k candidates can be extended by one token each;
- rank these on an estimate of their future success as complete responses, and retain again only k.

The simplest estimate of future success is the probability of the (incomplete) sequence as it stands.

Implementations vary (see for example [24]), but that's the general idea.

#### Beam Search Curse

With more computation (i.e., larger *k*), beam search reduces search errors (i.e., it finds responses with higher probabilities than greedy search does), but this does not always translate to better translations [aka 'the beam search curse'; 18].



A spoiler for this afternoon's lab

## The Length 'Culprit'

As beam size increases, and quality deteriorates, we often observe that the MAP decoder returns **shorter sequences** [33].

This observation led to various attempts at identifying a built-in bias towards short sequences and correct for it [15, 28, 36].

## **Controlling Length**

We can augment the MAP decoder with the ability to judge outcomes on their *length* besides their probabilities:

· length normalisation

$$\operatorname{argmax}_{c \in \mathcal{Y}} \frac{1}{|c|} \log p(c|x)$$

· length penalty

$$\operatorname{argmax}_{c \in \mathcal{Y}} \log p(c|x) - |c|\lambda$$

amongst others[2, 12, 15, 17, 28, 36]



### Regularised Beam Search

Meister et al. [23] views the 'search errors' of beam search as implicit (but interpretable) biases in search. They then express these biases explicitly as 'regularisers' on the original objective

$$\operatorname{argmax}_{c \in \mathcal{Y}} \log p(c|x) - \lambda \mathcal{R}(c, p_{\theta}(.|x))$$
 (5)

and use this framework to propose novel decoding strategies.

## Critical Eye

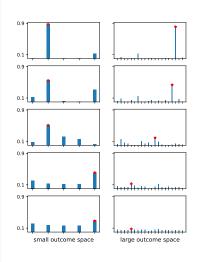
Remember my two contentions: i) go about finding the mode, and ii) what if the mode is of no special significance?

In relation to (ii)

- Stahlberg and Byrne [34] show that modes are often inadequate translations (such as the empty sequence);
- Eikema and Aziz [5] show that the mode is indeed often simply rare;
- · adequate samples (e.g., references) tend not to be modes [5, 25].

There's growing evidence that 'typically realisable' samples from autoregressive models exhibit a concentration of surprisal. Roughly, if models were efficient data stores, they would store adequate responses in samples of 'average surprisal'.

### Intuitions about the Mode

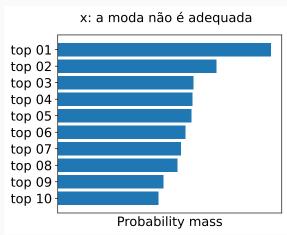


Our intuitions about modes quickly fall apart as outcome spaces grow very large.

Remember, the distribution over responses has infinitely many outcomes in it.

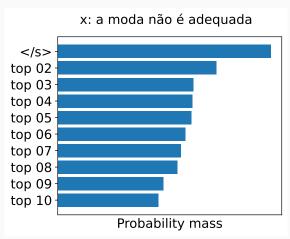
### Let's Develop Better Intuitions

This is how a MAP decoder makes decisions: it judges outcomes on probability alone.



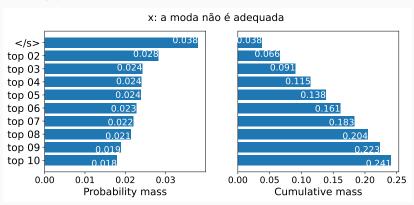
## Let's Develop Better Intuitions

But then the mode can be clearly inadequate



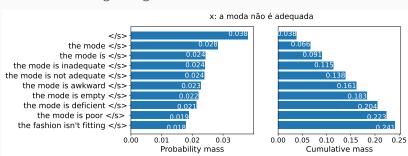
### Let's Develop Better Intuitions

#### But empty modes are often rare



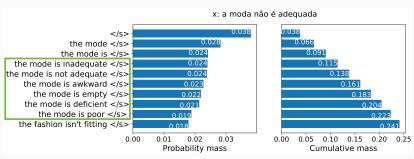
#### **Outcomes Matter!**

#### We have been neglecting the actual outcomes



### **Equivalence Classes**

The fact that every single outcome is rare does not mean the distribution codes no useful knowledge.



For example, Ilia and Aziz [16] use an external classifier to form such a class.

## **Principles Recap**

It's been a while... do you still remember the principles we outlined for decoding algorithms?

- translations should be in some sense preferred by the model (else, what is the difference between using one model or another?);
- 2. translations are ideally good for their prompts, let's say they ought to be *adequate*;

We came up with a number of ways to operationalise (1), but, with the exception of some pressure against awkwardly short outcomes, we barely considered (2).

### **Quality Estimate**

A 'quality estimate'  $\mu(c;x)$  quantifies the goodness of fit of a candidate translation c to the prompt x.

Examples: Cometkiwi [31]; average next-token surprisal (TP), average entropy (Softmax-Ent), inter alia [9].

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Outside MT, a function of this kind is better known as a reward model.

# **Re-Ranking**

Quality-aware approaches [8] are re-rankers, which typically work like this:

- enumerate a candidate set (e.g., using beam search or a sampler);
- 2. rank the candidates using a quality estimate (e.g., COMETKIWI);

$$y^{\text{decision}} = \operatorname{argmax}_{c \in \mathcal{Y}} \ \mu(c; x)$$
 (6)

## Critical Eye

The problem here is that, unless we are very careful, we violate principle 1. As the candidate list grows, the quality estimate will render the model less and less relevant.

In practice, this appears to be of no importance, after all, we are unlikely to enumerate too many candidates anyway (it's a costly operation). But, how so?

- If we were sampling, small sample size means riskier decisions;
- If we were already optimising a robust criterion, then why bother with quality estimation?

## Utility

Let's get back to quality of a translation, but we call it *utility*. Unlike quality, utility is a paired judgement.

We say that u(c, y; x) quantifies the benefit in choosing c as the translation of x when y is known to be a plausible translation of it.

Examples: human judgement, ChrF [29], BLEURT [32], COMET [30], etc.

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Outside MT, the utility u(c, y; x) is known as a paired reward.

### **Expected Utility**

We can design a 'quality estimate' by combining our LM with a utility function u(c, r; x) that compares a candidate translation c to a reference translation r.

In decoding, we do not have access to references, but in good probabilistic fashion, we can treat it as a *random variable* whose distribution our LM is assumed to predict from x.

We can then associate the merit of a candidate *c* with its *expected* utility under the model:

$$\mu_{\theta}(c;x) = \mathbb{E}_{p_{\theta}}[u(c,Y;x)] = \sum_{y \in \mathcal{Y}} p_{\theta}(y|x)u(c,y;x)$$
 (7)

### **Expected Utility - Example**

We derive a model-based notion of quality by computing a candidate's ChrF in expectation under the model (that is, using the model in place of a reference generator):

c	у	p(y x)	u(c, y;x)	p(y x) * u(c, y;x)
		0.0380	100.00	3.80
	the mode	0.0283	29.71	0.84
	the mode is	0.0242	24.93	0.60
	the mode is inadequate	0.0240	13.84	0.33
	the mode is not adequate	0.0238	13.25	0.32
	the mode is awkward	0.0227	15.97	0.36
	the mode is empty	0.0220	17.79	0.39
	the mode is deficient	0.0214	14.48	0.31
	the mode is poor	0.0189	18.87	0.36
	the fashion isn't fitting  []	0.0179	12.21	0.22
	[SUM]			24.68
the mode isn't adequate		0.0380	37.93	1.44
	the mode	0.0283	58.62	1.66
	the mode is	0.0242	62.16	1.51
	the mode is inadequate	0.0240	77.17	1.85
	the mode is not adequate	0.0238	82.98	1.98
	the mode is awkward	0.0227	45.80	1.04
	the mode is empty	0.0220	49.20	1.08
	the mode is deficient	0.0214	44.47	0.95
	the mode is poor	0.0189	49.81	0.94
	the fashion isn't fitting  []	0.0179	23.08	0.41
	[SUM]			36.18

## Maximisation of Expected Utility

Under the assumption that expected utility

$$\mu_{\theta}(c;x) = \mathbb{E}_{p_{\theta}}[u(c,Y;x)] = \sum_{y \in \mathcal{Y}} p_{\theta}(y|x)u(c,y;x)$$
 (8)

quantifies a reasonable notion of 'the quality of a candidate *c* in relation to a prompt *x*', we can use it for decision making:

$$y^{\text{MBR}} = \operatorname{argmax}_{c \in \mathcal{Y}} \ \mu_{\theta}(c; x) \ . \tag{9}$$

This is known as minimum Bayes risk decoding [21].

## Sampling-Based MBR

Eikema and Aziz [6] approximate expected utility using unbiased sampling

$$\mu_{\theta}(c;x) = \mathbb{E}_{p_{\theta}}[u(c,Y;x)] \stackrel{\text{MC}}{\approx} \frac{1}{S} \sum_{s=1}^{S} u(c,y^{(s)};x) \quad \text{where } y^{(s)} \sim p_{\theta}(.|x)$$

$$\tag{10}$$

Then they consider a reduced search space, made of N candidates  $c^{(1)}, \ldots, c^{(N)}$  enumerated via sampling (unbiased, biased) and/or beam search.

# Sampling-Based MBR Example

С	y ~ p(. x)	u(c, y;x)
	the mode is a mode	17.79
	is	58.01
	uncool	32.88
	the mode is awkward	15.97
	well I told you so didn't I ?	12.21
	fashionable	21.48
	the is	36.82
	the mode is poor	18.87
	mode is not cool	18.87
	the mode is very probable	12.71
	rare rare rare !	15.19
	mode is a mode	21.48
	I told you so didn't I ?	14.48
	nada nada	27.11
	mode is not cool	18.87
	the mode is inadequate	13.84
	aren't adequate	17.79
	sometimes NMT does strange things	9.59
	mode is weird	21.48
	the fashion isn't fitting	12.21
	[AVG]	20.88
the mode isn't adequate	mode	41.02
	nada nada nada	13.29
	modes aren't adequate	69.07
	the mode is a mode	55.00
	what ?	21.01
	mode mode mode	28.24
	the mode is actually rare	42.80
	the mode is a mode	55.00
	modes aren't adequate	69.07
	what ?	21.01
	the mode is poor	49.81
	the mode is deficient	44.47
	the the the the the	17.52
	the mode is	62.16
	nada nada nada	13.29
	mode is weird	35.96
	is the	35.67
	weird mode	33.37
	is	25.28
	is	25.28

### Critical Eye

MBR exploits similarity between responses to redistribute beliefs (can be thought of as a 'soft' way to form equivalence classes).

Less bias towards short translations, robustness to copying noise and hallucination [27]. Surprisal closer to that of references [25]. Improves substantially with modern neural utilities [10].

The search problem is formulated as re-ranking (expected utility does not bias the candidate set).

#### Learn to Search for MBR

To address the problem of approximate, incremental search for MBR, we have to address the problem of predicting expected rewards from incomplete responses [Monte Carlo Tree Search; 22].

Tomani et al. [35] formulated an approximation to this by training a model to perform quality estimation in addition to translation.

# **Origin Story**

Consider the 'exact match' utility u(c, y; x), which assigns 1 to c when it is identical to y.

It can be shown that

$$\mu_{\theta}(c; x) = \mathbb{E}_{p_{\theta}}[u(c, Y; x)] = p_{\theta}(c|x) \tag{11}$$

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which is mode-seeking (MAP) decoding.

The mode is the MBR solution using an arguably poor (low coverage) notion of utility.

#### Summary

To obtain some form of 'guarantee' for the one response we want to regard as 'the translation' of *x*, we turned away from sampling and towards *decision rules*.

The most probable translation (MAP decoding) ruled supreme for years, despite piling evidence against it.

Re-ranking enables the use of complex reward models, but at the expense of integration with the underlying MT model.

To meet both principles (that the output should be informed by the model and adequate) we can combine our model and a (paired) reward model, deriving MBR decoding.

MBR decoding is a class of objectives, and provides a strong rationale against MAP decoding.

Modern Decoding, as I see it

Modern training is a rather heterogenous combination of ideas:

1. we pretrain on 'all-we-can-eat' data

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- 1. we pretrain on 'all-we-can-eat' data
- 2. we then train on translation data, but also many other tasks
- we use a lot of synthetic data for example, to learn certain 'skills' (like in-context learning or chain-of-thought reasoning)
- 4. we learn from preference data (using RLHF or DPO, or whatnot)

In some of these steps we rely on 'samples' (e.g., 3 and 4), but this usually means biased samples with heterogenous (possibly undisclosed) hyperparameters.

#### The Probabilistic View

It's getting hard to insist in 'coherence with a certain probabilistic view' of the model, because this view is itself losing coherence.

That is okay, all this means is that 'the principled choice' argument, which was already weak, is now practically void of meaning.

This is good, it forces us to seek stronger rationales for our choices.

#### **General Advice**

If I am pressed to choose, here are some of my choices

- seek to establish equivalence classes (that is, exploit the fact that outcomes aren't linguistically unrelated to one another [20])
- or to, at least, incorporate similarity in scoring (e.g., softly like MBR and others [3] do)
- use a sampler to parameterise a decision rule but realise that due to heterogenous training, no sampler is privileged (we need to validate their properties in each model/data setting [11])

#### **Open Problems**

Efficient ways to search with non-factorised objectives (we mostly use re-ranking-type algorithms because it's hard to search efficiently, but re-ranking isn't that efficient either).

Decision rules for long-form generation (samplers claim most territory because we lack good decision rules).

Thanks!

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